Querying Inconsistent Databases

[Some] Past Research and Future Challenges

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University of Mons

DBDBD, Ghent, December 21, 2023

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Inconsistent Data

Perrey Reeves

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From Wikipedia, the free encyclopedia

Perrey Reeves (born 1970 or 1971 (age 52–53))[1] is an American film and television actress. She is best known for her recurring role as Melissa Gold on the television series *Entourage* from 2004 to 2011 and Marissa Jones in the 2003 comedy *Old School*.

Perrey Reeves

Early life [edit]

Reeves was born in New York City and raised in New Hampshire,[2] the daughter of Dr. Alexander Reeves, a

Inconsistent Databases

ACTORS	Name	Gender	Age
	Jolie	F	48
	Pitt	M	59
	Pitt	M	60

Every actor has, at most, one gender and one age:

`ACTORS PRIMARY KEY(Name).`

Data cleaning takes time (and money). Can we already obtain "reliable" information by querying the inconsistent database?

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Querying Inconsistent Databases

For ease of presentation, all queries return a Boolean (true/false).

ACTORS

Name	Gender	Age
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Pitt	M	59
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- Is Pitt's age 60?

$\exists y\,(\text{ACTORS}(\underline{\text{Pitt}}, y, 60))$ is "possibly false".

- Is Pitt older than Jolie?

$$\exists y \exists z \exists v \exists w \left(\begin{array}{l} \text{ACTORS}(\underline{\text{Pitt}}, y, z) \wedge \\ \text{ACTORS}(\underline{\text{Jolie}}, v, w) \wedge z > w \end{array} \right)$$ is "certainly true".

A block is a maximal set of tuples of the same relation that agree on their primary key (blocks are separated by dashed lines).
A repair (or possible world) is obtained by picking a single tuple from each block.

With this notion, "certainly true" means "true in every repair".

If 2 ages are stored for n actors, there are at least 2^n repairs.

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Consistent Query Answering for Primary Keys

Given a Boolean query Q, define the following decision problem:

Problem CERTAINTY(Q)

Input: A database instance that may violate primary-key constraints.

Question: Is Q true in every repair?

Example

If $Q_{60} = \exists y\,(\mathrm{ACTORS}(\underline{\mathrm{Pitt}}, y, 60))$, then the answer to CERTAINTY(Q_{60}) is "no" on our example database.

Remark

We assume that each relation name has a fixed primary key. Primary-key positions will be underlined. Primary keys can thus be derived from the query.

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Solving CERTAINTY(Q)

- A general solution in exponential time:

  Input: a database D
  for *each repair R of D* **do**
    if *Q is false in R* **then**
      return "no" (and halt)
  return "yes"

  ☞ CERTAINTY(Q) is in coNP for first-order queries Q.

- A smarter solution for $Q_{60} = \exists y\,(\text{ACTORS}(\underline{\text{Pitt}}, y, 60))$:

  Input: a database D
  Let $\overline{Q}_{60} := \exists y \exists z\,(\text{ACTORS}(\underline{\text{Pitt}}, y, z) \land \neg(z = 60))$
  if *Q_{60} is true and $\overline{Q}_{60}$ is false in D* **then**
    return "yes"
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SQL Rewriting

```
SELECT ‘yes’
FROM ACTORS
WHERE Name = ‘Pitt’
AND Age = 60;
```

$\rightsquigarrow$

```
SELECT ‘yes’
FROM ACTORS
WHERE Name = ‘Pitt’
AND Age = 60
AND NOT EXISTS (SELECT *
                FROM ACTORS
                WHERE Name = ‘Pitt’
                AND Age <> 60);
```

The Good, the Bad and the Ugly

Theorem (DBDBD, 2023)

For $Q_{\text{good}} = \exists y\,(\text{ACTORS}(\underline{\text{Pitt}}, y, 60))$, *the decision problem* CERTAINTY(Q_{good}) *is in* FO.

Theorem ([W., 2010])

For $Q_{\text{bad}} = \exists x \exists y\,(R(\underline{x}, y) \wedge S(\underline{y}, x))$, *the decision problem* CERTAINTY(Q_{bad}) *is in* P $\setminus$ FO.

P is the class of decision problems solvable in polynomial time.

Theorem ([Chomicki and Marcinkowski, 2005])

For $Q_{\text{ugly}} = \exists x_1 \exists x_2 \exists z\,(\text{ACTORS}(\underline{x_1}, \text{M}, z) \wedge \text{ACTORS}(\underline{x_2}, \text{F}, z))$, *the decision problem* CERTAINTY(Q_{ugly}) *is* coNP*-complete.*

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Research Agenda

- We aim to go beyond the task of determining CERTAINTY(Q) for individual queries Q.
- For "reasonable" classes $\mathcal{C}$ of queries, write an algorithm for the following problem:

Complexity Classification Task

Input: A query Q in the class $\mathcal{C}$.

Task: The computational complexity of CERTAINTY(Q), in terms of complexity classes like FO, P, coNP-complete,...

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Which Query Classes Are "Reasonable"?

- The class of (Boolean) conjunctive queries (a.k.a. Select-Project-Join queries):

$$\exists \vec{u} \left(R_1(\underline{\vec{x}_1}, \vec{y}_1) \land R_2(\underline{\vec{x}_2}, \vec{y}_2) \land \cdots \land R_n(\underline{\vec{x}_n}, \vec{y}_n) \right). \qquad (1)$$

- The class of disjunctions of conjunctive queries (a.k.a. UCQ queries):

$$Q_1 \lor Q_2 \lor \cdots \lor Q_m,$$

where each Q_i is of the form (1).

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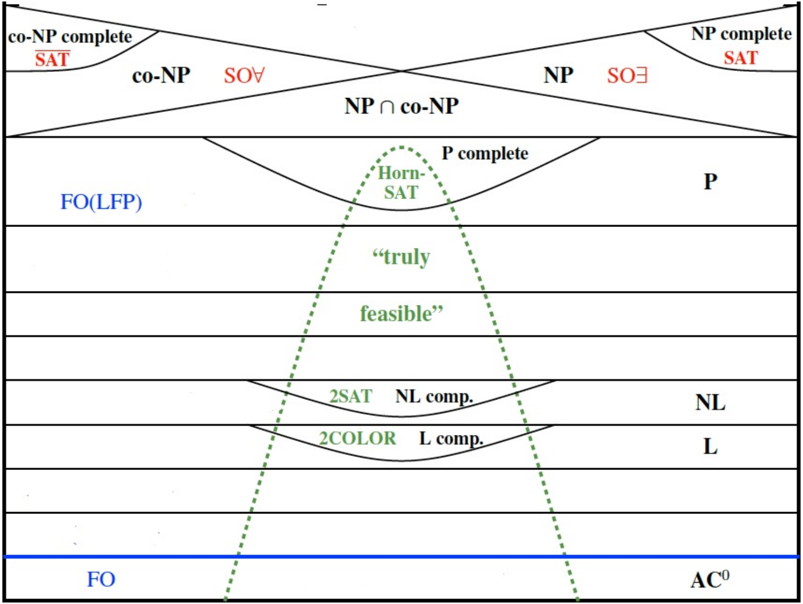

Classifying CERTAINTY(Q) in P/coNP-complete is Hard

Recall

If P $\neq$ coNP, then some problems in coNP are neither in P nor coNP-complete.

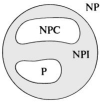

Figure 7.1 The world of NP, reprised (assuming P$\neq$NP).

Conjecture

If Q is a disjunction of conjunctive queries, then CERTAINTY(Q) *is in* P *or* coNP*-complete.*

Theorem ([Fontaine, 2015])

The above conjecture implies Bulatov's dichotomy theorem for the conservative constraint satisfaction problem (CSP).

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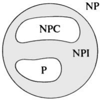

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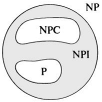

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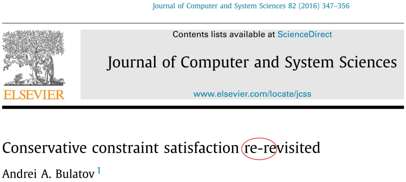
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Conservative constraint satisfaction re-revisited

Andrei A. Bulatov [1]

Is it Easier for Conjunctive Queries?

Conjecture

If Q is of the form $\exists \vec{u} \left(R_1(\underline{\vec{x}_1}, \vec{y}_1) \wedge \cdots \wedge R_n(\underline{\vec{x}_n}, \vec{y}_n) \right)$, then CERTAINTY($Q$) *is in* P *or* coNP*-complete.*

Theorem ([Koutris and W., 2017])

The above conjecture holds under the assumption that $R_i \neq R_j$ whenever $i \neq j$.

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The Good Among The Good, the Bad and the Ugly

A directed graph, called attack graph, is defined for every conjunctive query.

Theorem ([Koutris and W., 2017])

Let $Q = \exists \vec{u} \left(R_1(\underline{\vec{x}_1}, \vec{y}_1) \wedge \cdots \wedge R_n(\underline{\vec{x}_n}, \vec{y}_n) \right)$ with $R_i \neq R_j$ for $i \neq j$. Then,

- *if Q's attack graph is acyclic, then* CERTAINTY(Q) *is in* FO;
- *if Q's attack graph is cyclic, then* CERTAINTY(Q) *is* L*-hard.*

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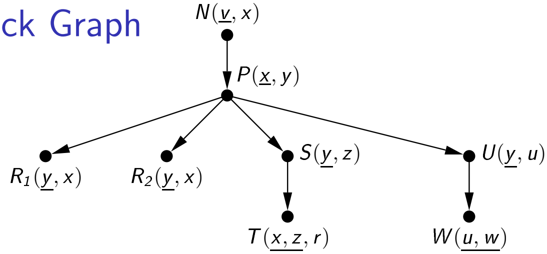
Attack Graph



$N^+ = \{v\}$

$P^+ = \{x\}$

${R_1}^+ = \{y, x, z, r, u\}$

${R_2}^+ = \{y, x, z, r, u\}$

$S^+ = \{y, x, u\}$

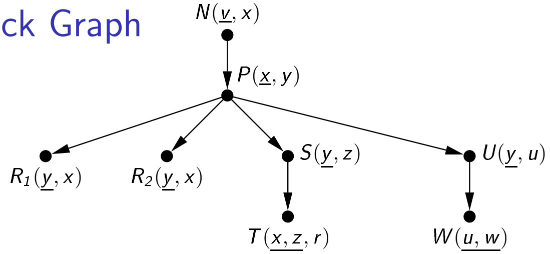
$U^+ = \{y, x, z, r\}$

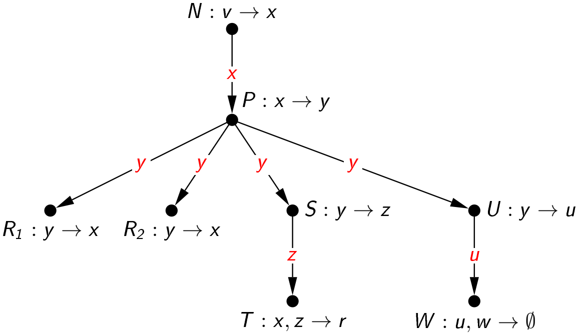
$T^+ = \{x, z, y, u\}$

$W^+ = \{u, w\}$

S^+, e.g., is the closure of S's key w.r.t. all other FDs.
S can attack with $z \notin S^+$.

Attack Graph





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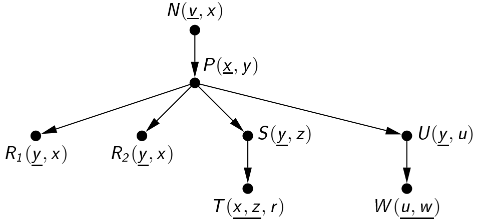
Attack Graph and (Consistent) First-Order Rewriting

Q :



We construct a first-order formula φ_N such that for every database:

φ_N is true in the database $\iff$ Q is true in every repair.

$$\begin{aligned}
\varphi_N &:= \exists v\,(\exists x\,(N(\underline{v}, x)) \wedge \neg\exists x\,(N(\underline{v}, x) \wedge \neg\varphi_P(x))) \\
\varphi_P(x) &:= \exists y\,(P(\underline{x}, y)) \\
&\quad \wedge \neg\exists y\,(P(\underline{x}, y) \wedge \neg(\varphi_{R_1}(x, y) \wedge \varphi_{R_2}(x, y) \wedge \varphi_S(x, y) \wedge \varphi_U(y))) \\
\varphi_{R_i}(x, y) &:= R_i(\underline{y}, x) \wedge \neg\exists x'\,(R_i(\underline{y}, x') \wedge x' \neq x),\ 1 \leq i \leq 2 \\
\varphi_S(x, y) &:= \exists z\,(S(\underline{y}, z)) \wedge \neg\exists z\,(S(\underline{y}, z) \wedge \neg\exists r\,(T(\underline{x, z}, r))) \\
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\end{aligned}$$

In SQL, we get 4 embedded NOT EXISTS...

Attack Graph and (Consistent) First-Order Rewriting

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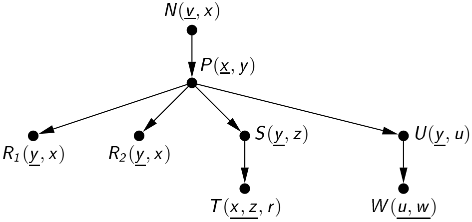

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$$\varphi_P(x) := \exists y\,(P(\underline{x}, y)) \land \neg\exists y\,(P(\underline{x}, y) \land \neg(\varphi_{R_1}(x, y) \land \varphi_{R_2}(x, y) \land \varphi_S(x, y) \land \varphi_U(y)))$$

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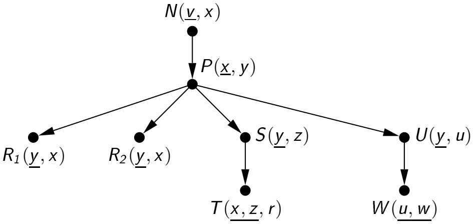

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$$\land \neg\exists y \left(P(\underline{x}, y) \land \neg\left(\varphi_{R_1}(x, y) \land \varphi_{R_2}(x, y) \land \varphi_S(x, y) \land \varphi_U(y)\right)\right)$$

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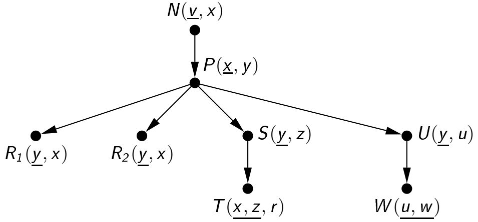
In SQL, we get 4 embedded NOT EXISTS...

Attack Graph and (Consistent) First-Order Rewriting

Q :



We construct a first-order formula φ_N such that for every database:

φ_N is true in the database $\iff$ Q is true in every repair.

$$\begin{aligned}
\varphi_N &:= \exists v\,(\exists x\,(N(\underline{v}, x)) \wedge \neg\exists x\,(N(\underline{v}, x) \wedge \neg\varphi_P(x)))\\
\varphi_P(x) &:= \exists y\,(P(\underline{x}, y))\\
&\quad \wedge \neg\exists y\,(P(\underline{x}, y) \wedge \neg(\varphi_{R_1}(x, y) \wedge \varphi_{R_2}(x, y) \wedge \varphi_S(x, y) \wedge \varphi_U(y)))\\
\varphi_{R_i}(x, y) &:= R_i(\underline{y}, x) \wedge \neg\exists x'\left(R_i(\underline{y}, x') \wedge x' \neq x\right), 1 \leq i \leq 2\\
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Attack Graph and (Consistent) First-Order Rewriting

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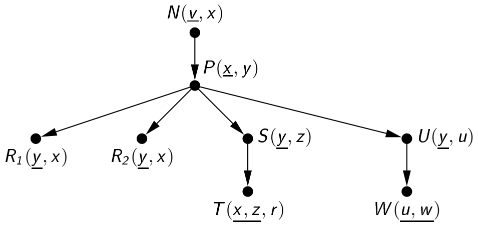

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&\quad \wedge \neg \exists y \, (P(\underline{x}, y) \wedge \neg (\varphi_{R_1}(x, y) \wedge \varphi_{R_2}(x, y) \wedge \varphi_S(x, y) \wedge \varphi_U(y))) \\
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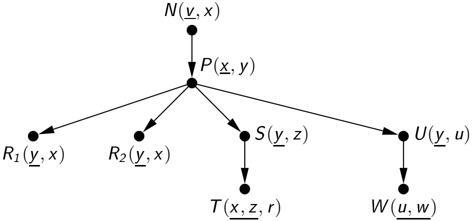

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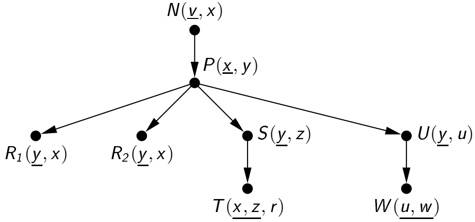

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Observation Regarding Correctness

$$\varphi_{R_1}(x,y) := R_1(\underline{y},x) \wedge \neg\exists x' \left(R_1(\underline{y},x') \wedge x' \neq x\right)$$
$$\varphi_{R_2}(x,y) := R_2(\underline{y},x) \wedge \neg\exists x' \left(R_2(\underline{y},x') \wedge x' \neq x\right)$$

In words,

- $\varphi_{R_1}(x,y)$: there is a singleton block containing $R_1(\underline{y},x)$;
- $\varphi_{R_2}(x,y)$: there is a singleton block containing $R_2(\underline{y},x)$.

That is, R_i-blocks of size ≥ 2 can be ignored. For example,

R_1	$\underline{y}$	x
	a	c_1
	a	c_2

R_2	$\underline{y}$	x
	a	c_1
	a	c_2

To construct a repair that falsifies the query, pick $R_1(\underline{a},c_i)$ and $R_2(\underline{a},c_j)$ such that $c_i \neq c_j$.

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R_2

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a	c_1
a	c_2

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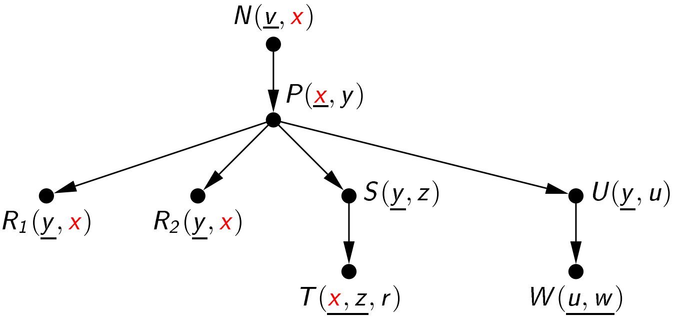
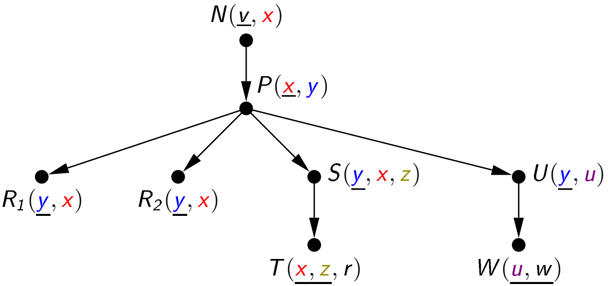
Attack Graph $\neq$ Join Tree



The subgraph induced by atoms that contain x is not connected.

Attack Graph that Is a Join Tree

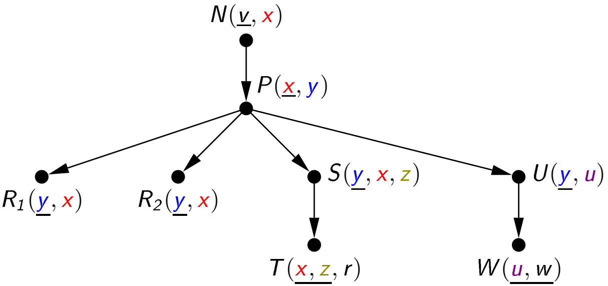


Moreover, every internal node V has zero indegree in the attack graph of the subquery rooted at V ($V \in \{P, S, U\}$).
Such a join tree is called a Pair-Pruning Join Tree (PPJT).
Yannakakis' algorithm extends to the inconsistent setting:

Theorem ([Fan et al., 2023])

If Q has a PPJT, then CERTAINTY(Q) *is in* LIN *(i.e., problems solvable in linear time).*

Attack Graph that Is a Join Tree



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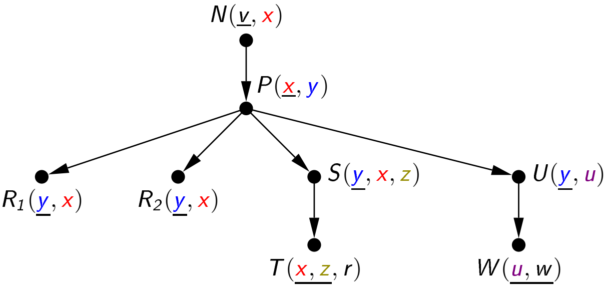

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Yannakakis+Pruning

$$Answer(\text{yes}) \leftarrow N(\underline{v}, x) \wedge \neg N^{\text{fadingkey}}(v)$$
$$N^{\text{fadingkey}}(v) \leftarrow N(\underline{v}, x) \wedge \neg P^{\text{join}}(x)$$

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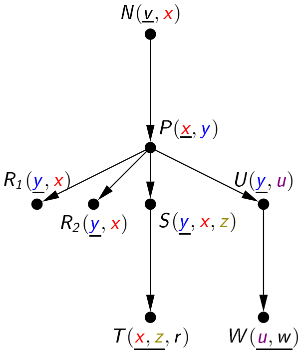
$$R_i^{\text{fadingkey}}(y) \leftarrow R_i(\underline{y}, x) \wedge R_i(\underline{y}, x') \wedge x \neq x'$$
$$(1 \leq i \leq 2)$$



$$T^{\text{join}}(x, z) \leftarrow T(\underline{x, z}, r)$$
$$W^{\text{join}}(u) \leftarrow W(\underline{u, w})$$

Yannakakis+Pruning

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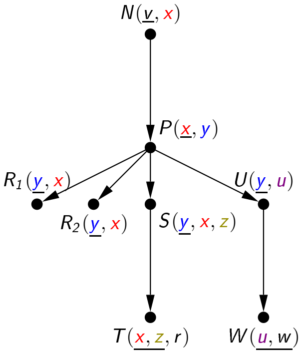

$$U^{\text{join}}(y) \leftarrow U(\underline{y}, u) \wedge \neg U^{\text{fadingkey}}(y)$$
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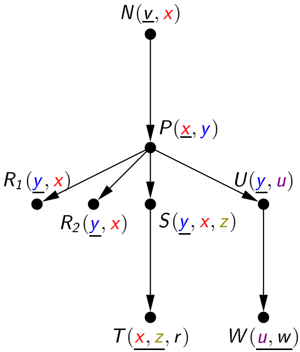

$$S^{\text{join}}(x, y) \leftarrow S(\underline{y}, x, z) \wedge \neg S^{\text{fadingkey}}(y)$$
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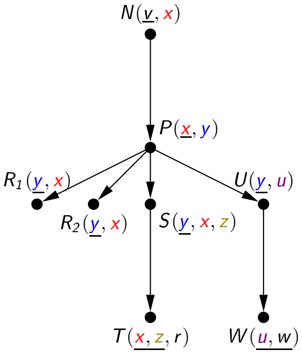

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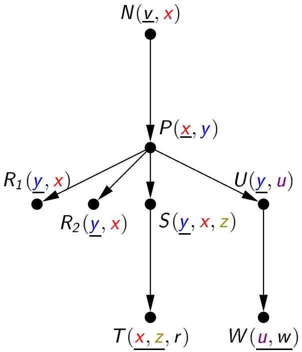

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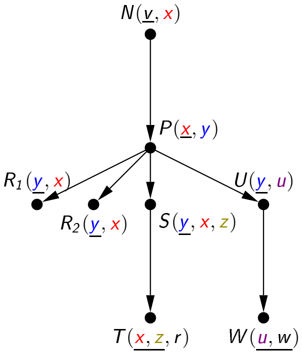

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LinCQA

- LinCQA is a system that takes as input any query with a PPJT and outputs rewritings in both SQL and non-recursive Datalog with negation.
- `https://github.com/xiatingouyang/LinCQA/`
- See [Fan et al., 2023] for experiments.

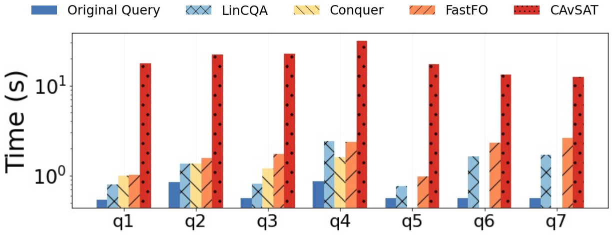

Table of Contents

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Range Consistent Query Answering [Arenas et al., 2001]

For queries returning numbers instead of Booleans.
For ease of presentation, all queries return a single number.

MOVIES

Title	Actor
Mr. & Mrs. Smith	Jolie
Mr. & Mrs. Smith	Pitt
Mr. & Mrs. Smith	Reeves

ACTORS

Name	Gender	Age
Jolie	F	48
Pitt	F	59
Pitt	M	60
Reeves	F	52
Reeves	F	53

Get the sum of ages of all actresses in Mr. & Mrs. Smith:

$$\mathtt{SUM}(z) \leftarrow \text{MOVIES}(\underline{\text{Mr. \& Mrs. Smith}, x}), \text{ACTORS}(\underline{x}, \text{F}, z).$$

- The lowest answer across all repairs is $48 + 52 = 100$;
- the greatest answer across all repairs is $48 + 59 + 53 = 160$;
- the interval $[100, 160]$ is called the range consistent answer.

Range Consistent Query Answering [Arenas et al., 2001]

For queries returning numbers instead of Booleans.
For ease of presentation, all queries return a single number.

MOVIES

<u>Title</u>	<u>Actor</u>
Mr. & Mrs. Smith	Jolie
Mr. & Mrs. Smith	Pitt
Mr. & Mrs. Smith	Reeves

ACTORS

<u>Name</u>	Gender	Age
Jolie	F	48
Pitt	F	59
Pitt	M	60
Reeves	F	52
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Formal Setting

- Numerical terms $f()$ expressible in the (safe) rule format

$$\texttt{AGG}(r) \leftarrow R_1(\underline{\vec{x}_1}, \vec{y}_1) \wedge R_2(\underline{\vec{x}_2}, \vec{y}_2) \wedge \cdots \wedge R_n(\underline{\vec{x}_n}, \vec{y}_n), \qquad (2)$$

  where r is either a numerical variable or a constant, and AGG is an aggregate operator (e.g., MAX, MIN, SUM, COUNT, AVG).
- Given a database instance, let $f^+()$ and $f^-()$ be, respectively, the greatest and smallest values of $f()$ across all repairs.
- Aggregate logic = first-order logic + aggregate operators.
- When can $f^+()$ and $f^-()$ be expressed in aggregate logic?
  - Not investigated since [Fuxman, 2007].
  - It is easily shown that $f^+()$ and $f^-()$ are not expressible in aggregate logic if the attack graph of the body of (2) has a cycle.
  - Does the converse hold?

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Rewriting Example

$$\mathtt{SUM}(z) \leftarrow \mathsf{MOVIES}(\underline{\mathsf{Mr.\ \&\ Mrs.\ Smith}, x}), \mathsf{ACTORS}(\underline{x}, \mathsf{F}, z).$$

- Upper bound rewriting:

$$\mathsf{U}(x, \mathtt{MAX}(z)) \leftarrow \mathsf{MOVIES}(\underline{\mathsf{Mr.\ \&\ Mrs.\ Smith}, x}), \mathsf{ACTORS}(\underline{x}, \mathsf{F}, z)$$
$$\mathsf{UB}(\mathtt{SUM}(z)) \leftarrow \mathsf{U}(x, z)$$

- Lower bound rewriting:

$$\mathsf{POSSIBLE\_M}(x) \leftarrow \mathsf{ACTORS}(\underline{x}, \mathsf{M}, z)$$
$$\mathsf{CERTAIN\_F}(x, z) \leftarrow \mathsf{ACTORS}(\underline{x}, \mathsf{F}, z), \neg \mathsf{POSSIBLE\_M}(x)$$
$$\mathsf{L}(x, \mathtt{MIN}(z)) \leftarrow \mathsf{MOVIES}(\underline{\mathsf{Mr.\ \&\ Mrs.\ Smith}, x}), \mathsf{CERTAIN\_F}(\underline{x}, z)$$
$$\mathsf{LB}(\mathtt{SUM}(z)) \leftarrow \mathsf{L}(x, z)$$

Table of Contents

Counting

Given a Boolean query Q, define the following counting problem:

Problem $\sharp$CERTAINTY(Q)

Input: A database instance that may violate primary-key constraints.

Question: How many repairs of satisfy Q?

Complexity Classification Task

Input: A self-join-free Boolean conjunctive query Q.

Task: Determine lower and upper complexity bounds on the complexity of $\sharp$CERTAINTY(q), in terms of common complexity classes like FP and $\sharp$P.

- Solved in [Maslowski and W., 2013] and generalized to FDs in [Calautti et al., 2022].
- Same problem as query answering in block-independent disjoint (BID) probabilistic databases under the restriction that in every block $\mathbf{b}$, every tuple has probability $\frac{1}{|\mathbf{b}|}$.

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BID Databases

Every input to CERTAINTY(Q) is a block-independent disjoint database without probabilities (or with uniform probabilities).

☞ Inconsistency is not only a burden, but also a chance. [1]

Researchers:

	Name	Affiliation	P
t_1^1	Fred	U. Washington	$p_1^1 = 0.3$
t_1^2		U. Wisconsin	$p_1^2 = 0.2$
t_1^3		Y! Research	$p_1^3 = 0.5$
t_2^1	Sue	U. Washington	$p_2^1 = 1.0$
t_3^1	John	U. Wisconsin	$p_3^1 = 0.7$
t_3^2		U. Washington	$p_3^2 = 0.3$
t_4^1	Frank	Y! Research	$p_4^1 = 0.9$
t_4^2		M. Research	$p_4^2 = 0.1$

[1] Inspired by [Kern-Isberner and Lukasiewicz, 2017]. The image is from [Dalvi et al., 2009].

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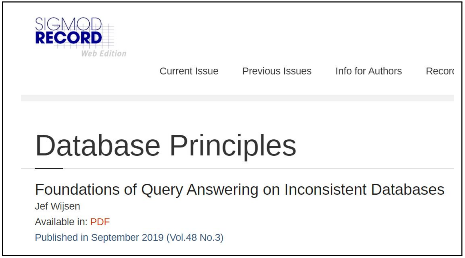
Concluding Remarks

Consistent Query Answering is an active research area since [Arenas et al., 1999]:

- Database repairing w.r.t. different classes of constraints
- Database repairing and data exchange
- Database repairing and approximations
- Database repairing and preferences
- Database repairing and implementations
- Database repairing and database management systems
- Consistent query answering for queries with negation
- Consistent query answering in description logics
- Consistent query answering over graph databases
- ...



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Database Principles

Foundations of Query Answering on Inconsistent Databases

Jef Wijsen

Available in: PDF

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Thanks!

FYI, Brad Pitt celebrated his 60th birthday on December 18, 2023.

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