## Consistent Query Answering with Respect to Primary Keys [Some] Past Research and Future Challenges

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Complexity of CERTAINTY(Q)

CERTAINTY(Q) in Linear Time (and in FO)

Alternative Semantics

**Concluding Remarks** 

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#### Motivation

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## Inconsistent Data

≡ Perrey Reeves			文 <sub>人</sub> 18 Ian
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From Wikipedia, the free encyclopedia			
Perrey Reeves (born 1970 or 1971 (age 52–53) <sup>11</sup> is an American film and television actross. She is best known for her recurring role as Melissa Gold on the television series <i>Entourage</i> from 2004 to 2011 and Marissa Jones in the 2003 comedy <i>Old School</i> .	2	Perre	ey Reeves
Reeves was born in New York City and raised in New Hampshire, <sup>[2]</sup> the daughter of Dr. Alexander Reeves, a			٣.

### Inconsistent Databases

ACTORS	<u>Name</u>	Gender	Age
	Jolie	F	48
	Pitt	M	59
	Pitt	М	60

Every actor has, at most, one gender and one age: ACTORS PRIMARY KEY(Name).

Data cleaning takes time (and money). Can we already obtain "reliable" information by querying the inconsistent database?

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## Querying Inconsistent Databases

For ease of presentation,	ACTORS	<u>Name</u>	Gender	Age
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Boolean (true/false).		Pitt	M	59
		Pitt	М	60

Is Pitt's age 60?

 $\exists y (ACTORS(\underline{Pitt}, y, 60)) \text{ is "possibly false"}.$ 

#### Is Pitt older than Jolie?

$$\exists y \exists z \exists v \exists w \left( \begin{array}{c} \mathsf{ACTORS}(\underline{\mathsf{Pitt}}, y, z) \land \\ \mathsf{ACTORS}(\underline{\mathsf{Jolie}}, v, w) \land z > w \end{array} \right) \text{ is "certainly true"}$$

A block is a maximal set of tuples of the same relation that agree on their primary key (blocks are separated by dashed lines). A repair (or possible world) is obtained by picking a single tuple from each block.

With this notion, "certainly true" means "true in every repair". If 2 ages are stored for *n* actors, there are at least 2<sup>*n*</sup> repairs.

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## Consistent Query Answering for Primary Keys

Given a Boolean query Q, define the following decision problem:

#### Problem CERTAINTY(Q)

Input: A database instance *D* that may violate primary-key constraints.

Question: Is Q true in every repair of D?

#### Example

If  $Q_{60} = \exists y (ACTORS(Pitt, y, 60))$ , then the answer to CERTAINTY( $Q_{60}$ ) is "no" on our example database.

#### Remark

We assume that each relation name has a fixed primary key. Primary-key positions will be underlined. Primary keys can thus be derived from the query.

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## Solving CERTAINTY(Q)

Proposition CERTAINTY(Q) is in coNP for first-order queries Q. Proof. A "'no" certificate is a repair that falsifies Q.

CERTAINTY( $Q_{60}$ ) is in FO, as the following are equivalent for every database instance D:

- 1. Q is true in every repair of D;
- 2. *D* satisfies  $Q_{60} \land \neg \exists y \exists z (ACTORS(\underline{Pitt}, y, z) \land (z \neq 60)).$

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Proposition

For  $Q_{good} = \exists y (ACTORS(\underline{Pitt}, y, 60))$ , the decision problem CERTAINTY( $Q_{good}$ ) is in FO.

Theorem ([W., 2010])

For  $Q_{bad} = \exists x \exists y (R(\underline{x}, y) \land S(\underline{y}, x))$ , the decision problem CERTAINTY( $Q_{bad}$ ) is in P \ FO (later, it was proven L-complete).

Theorem ([Chomicki and Marcinkowski, 2005]) For  $Q_{ugly} = \exists x_1 \exists x_2 \exists z (ACTORS(x_1, M, z) \land ACTORS(x_2, F, z)),$ the decision problem CERTAINTY( $Q_{ugly}$ ) is coNP-complete.



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## Research Agenda

We aim to go beyond the task of determining CERTAINTY(Q) for individual queries Q.

► For "reasonable" classes C of queries, write an algorithm for the following problem:

Complexity Classification Task Input: A query Q in the class C. Task: The computational complexity of CERTAINTY(Q), in terms of complexity classes like FO, P, coNP-complete,...

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Which Query Classes Are "Reasonable"?

The class of (Boolean) conjunctive queries (a.k.a. Select-Project-Join queries):

$$\exists \vec{u} \left( R_1(\underline{\vec{x_1}}, \vec{y_1}) \land R_2(\underline{\vec{x_2}}, \vec{y_2}) \land \dots \land R_n(\underline{\vec{x_n}}, \vec{y_n}) \right).$$
(1)

The class of disjunctions of conjunctive queries (a.k.a. UCQ queries):

 $Q_1 \vee Q_2 \vee \cdots \vee Q_m,$ 

where each  $Q_i$  is of the form (1).

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## Which Complexity Classes?



## Classifying CERTAINTY(Q) in P/coNP-complete is Hard

#### Conjecture

If Q is a disjunction of conjunctive queries, then CERTAINTY(Q) is in P or coNP-complete.

### Theorem ([Fontaine, 2015])

The above conjecture implies Bulatov's dichotomy theorem for the conservative constraint satisfaction problem (CSP). Classifying CERTAINTY(Q) in P/coNP-complete is Hard

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Conservative constraint satisfaction re-revisited

#### Conjecture

If Q is of the form  $\exists \vec{u} (R_1(\vec{x_1}, \vec{y_1}) \land \cdots \land R_n(\vec{x_n}, \vec{y_n}))$ , then CERTAINTY(Q) is in P or coNP-complete.

## Theorem ([Koutris and W., 2017])

The above conjecture holds under the assumption that  $R_i \neq R_j$  whenever  $i \neq j$ .

Somewhat later, if was proven that for every self-join-free CQ Q, CERTAINTY(Q) is either in FO, L-complete, or coNP-complete.

#### Theorem ([Padmanabha et al., 2023])

The above conjecture holds under the assumption that n = 2.

#### Theorem ([Koutris et al., 2021])

The above conjecture holds for queries of the form  $\exists x_1 \cdots \exists x_{n+1} (R_1(\underline{x_1}, x_2) \land R_2(\underline{x_2}, x_3) \land \cdots \land R_n(\underline{x_n}, x_{n+1}))$ 

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## The Good Among the Good, the Bad and the Ugly

A directed graph, called attack graph, is defined for every conjunctive query.

Theorem ([Koutris and W., 2017]) Let  $Q = \exists \vec{u} \left( R_1(\underline{\vec{x_1}}, \vec{y_1}) \land \dots \land R_n(\underline{\vec{x_n}}, \vec{y_n}) \right)$  with  $R_i \neq R_j$  for  $i \neq j$ . Then,

• if Q's attack graph is acyclic, then CERTAINTY(Q) is in FO;

▶ if Q's attack graph is cyclic, then CERTAINTY(Q) is L-hard.

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$$N^{+} = \{v\}$$

$$P^{+} = \{x\}$$

$$R_{1}^{+} = \{y, x, z, r, u\}$$

$$R_{2}^{+} = \{y, x, z, r, u\}$$

$$S^{+} = \{y, x, u\}$$

$$U^{+} = \{y, x, z, r\}$$

$$T^{+} = \{x, z, y, u\}$$

$$W^{+} = \{u, w\}$$
er EDs

 $S^+$ , e.g., is the closure of S's key w.r.t. all other FDs. S can attack with  $z \notin S^+$ .



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## Attack Graph and (Consistent) First-Order Rewriting



We construct a first-order formula  $\varphi_N$  such that for every database:

 $\varphi_N$  is true in the database  $\iff Q$  is true in every repair.

 $\varphi_{N} := \exists v \left( \exists x \left( N(\underline{v}, x) \right) \land \neg \exists x \left( N(\underline{v}, x) \land \neg \varphi_{P}(v, x) \right) \right),$ 

where  $\varphi_P(v, x)$  is a rewriting of the conjunctive query whose atoms are the atoms of Q except  $N(\underline{v}, x)$ , in which variables vand x are free. The empty query rewrites to true

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Attack Graph  $\neq$  Join Tree



The subgraph induced by atoms that contain x is not connected.

## Attack Graph that Is a Join Tree



Moreover, every internal node V has zero indegree in the attack graph of the subquery rooted at V ( $V \in \{P, S, U\}$ ). Such a join tree is called a Pair-Pruning Join Tree (PPJT). Yannakakis' algorithm extends to the inconsistent setting:

#### Theorem ([Fan et al., 2023])

If Q has a PPJT, then CERTAINTY(Q) is in LIN (i.e., problems solvable in linear time).

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 $T^{\text{join}}(x, z) \leftarrow T(\underline{x, z}, r)$  $W^{\text{join}}(u) \leftarrow W(\underline{u, w})$ 

 $\begin{aligned} &Answer(\text{yes}) \leftarrow N(\underline{v}, x) \land \neg N^{\text{fadingkey}}(v) \\ &N^{\text{fadingkey}}(v) \leftarrow N(\underline{v}, x) \land \neg P^{\text{join}}(x) \end{aligned}$ 

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 $\begin{aligned} & \mathcal{P}^{\mathsf{join}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg \mathcal{P}^{\mathsf{fadingkey}}(x) \\ & \mathcal{P}^{\mathsf{fadingkey}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg \mathcal{U}^{\mathsf{join}}(y) \\ & \mathcal{P}^{\mathsf{fadingkey}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg S^{\mathsf{join}}(x, y) \\ & \mathcal{P}^{\mathsf{fadingkey}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg R_{i}^{\mathsf{join}}(x, y) \end{aligned}$ 

 $U^{\text{join}}(\mathbf{y}) \leftarrow U(\underline{\mathbf{y}}, u) \land \neg U^{\text{fadingkey}}(\mathbf{y})$  $U^{\text{fadingkey}}(\mathbf{y}) \leftarrow U(\underline{\mathbf{y}}, u) \land \neg W^{\text{join}}(u)$ 

$$\begin{split} S^{\mathsf{join}}(x,y) &\leftarrow S(\underline{y},x,z) \land \neg S^{\mathsf{fadingkey}}(y) \\ S^{\mathsf{fadingkey}}(y) &\leftarrow S(\underline{y},x,z) \land S(\underline{y},x',z) \land x \neq x' \\ S^{\mathsf{fadingkey}}(y) &\leftarrow S(\underline{y},x,z) \land \neg T^{\mathsf{join}}(x,z) \end{split}$$



 $T^{\text{join}}(x, z) \leftarrow T(\underline{x}, \underline{z}, r)$  $W^{\text{join}}(u) \leftarrow W(u, w)$ 

$$\begin{split} S^{\text{join}}(x, y) &\leftarrow S(\underline{y}, x, z) \land \neg S^{\text{fadingkey}}(y) \\ S^{\text{fadingkey}}(y) &\leftarrow S(\underline{y}, x, z) \land S(\underline{y}, x', z) \land x \neq x' \\ S^{\text{fadingkey}}(y) &\leftarrow S(\underline{y}, x, z) \land \neg T^{\text{join}}(x, z) \\ R^{\text{join}}_{i}(x, y) &\leftarrow R_{i}(\underline{y}, x) \land \neg R^{\text{fadingkey}}_{i}(y) \\ R^{\text{fadingkey}}_{i}(y) &\leftarrow R_{i}(\underline{y}, x) \land R_{i}(\underline{y}, x') \land x \neq x' \\ (1 \leq i \leq 2) \end{split}$$

 $\begin{aligned} &Answer(\text{yes}) \leftarrow N(\underline{v}, x) \land \neg N^{\text{fadingkey}}(v) \\ &N^{\text{fadingkey}}(v) \leftarrow N(\underline{v}, x) \land \neg P^{\text{join}}(x) \end{aligned}$ 

$$\begin{split} & \mathcal{P}^{\mathsf{join}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg \mathcal{P}^{\mathsf{fadingkey}}(x) \\ & \mathcal{P}^{\mathsf{fadingkey}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg \mathcal{U}^{\mathsf{join}}(y) \\ & \mathcal{P}^{\mathsf{fadingkey}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg S^{\mathsf{join}}(x, y) \\ & \mathcal{P}^{\mathsf{fadingkey}}(x) \leftarrow \mathcal{P}(\underline{x}, y) \land \neg R_{i}^{\mathsf{join}}(x, y) \end{split}$$

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 $T^{\text{join}}(x,z) \leftarrow T(x,z,r)$  $W^{\text{join}}(u) \leftarrow W(u, w)$ 

$$S^{\text{fadingkey}}(\underline{y}) \leftarrow S(\underline{y}, x, z) \land \neg S \land \forall \forall \forall y)$$

$$S^{\text{fadingkey}}(\underline{y}) \leftarrow S(\underline{y}, x, z) \land S(\underline{y}, x', z) \land x \neq x'$$

$$S^{\text{fadingkey}}(\underline{y}) \leftarrow S(\underline{y}, x, z) \land \neg T^{\text{join}}(x, z)$$

$$R^{\text{join}}_{i}(x, y) \leftarrow R_{i}(\underline{y}, x) \land \neg R^{\text{fadingkey}}_{i}(\underline{y})$$

$$R^{\text{fadingkey}}_{i}(\underline{y}) \leftarrow R_{i}(\underline{y}, x) \land R_{i}(\underline{y}, x') \land x \neq x'$$

$$(1 \leq i \leq 2)$$

$$22/38$$

$$P^{\text{join}}(x) \leftarrow P(\underline{x}, y) \land \neg P^{\text{fadingkey}}(x)$$

$$P^{\text{fadingkey}}(x) \leftarrow P(\underline{x}, y) \land \neg U^{\text{join}}(y)$$

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 $S_{join}(x, y) \neq S(y, x, z) \wedge S_{join}(x, y) \neq S_{join}(x, y) \neq S_{join}(x, y) \wedge S_{join}$ 



 $T^{\text{join}}(x, z) \leftarrow T(\underline{x}, \underline{z}, r)$  $W^{\text{join}}(u) \leftarrow W(u, w)$ 

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 $Answer(yes) \leftarrow N(\underline{v}, x) \land \neg N^{\mathsf{fadingkey}}(v)$ 

 $N^{\text{fadingkey}}(v) \leftarrow N(v, x) \land \neg P^{\text{join}}(x)$ 

## **Observation Regarding Correctness**

 $R_i$ -blocks of size  $\geq 2$  can be ignored. For example,

$R_1$	<u>y</u>	x	$R_2$	<u>y</u>	x
	а	<i>c</i> <sub>1</sub>		а	<i>c</i> <sub>1</sub>
	а	<i>c</i> <sub>2</sub>		а	_ <i>c</i> <sub>2</sub>

To construct a repair that falsifies the query, pick  $R_1(\underline{a}, c_i)$  and  $R_2(\underline{a}, c_j)$  such that  $c_i \neq c_j$ .

## LinCQA

- LinCQA is a system that takes as input any query with a PPJT and outputs rewritings in both SQL and non-recursive Datalog with negation.
- https://github.com/xiatingouyang/LinCQA/
- See [Fan et al., 2023] for experiments.



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## Range Consistent Query Answering [Arenas et al., 2001]

For queries returning numbers instead of Booleans.

For ease of presentation, all queries return a single number.



Get the age of the oldest actress:

 $MAX(z) \leftarrow ACTORS(\underline{x}, F, z).$ 

- The lowest answer across all repairs is  $MAX({48, 52}) = 52;$
- the greatest answer across all repairs is MAX({48,59,53}) = 59;
- ▶ the interval [52, 59] is called the range consistent answer

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4	CTORS		
	<u>Name</u>	Gender	Age
	Jolie	F	48
	Pitt	<u>F</u>	59
	Pitt	М	60
	Reeves	<u>-</u>	52
	Reeves	F	53
	Pitt Pitt Reeves Reeves	<mark>F</mark> M F F	- 59 60 - 52 53

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## Formal Setting

Numerical terms f() expressible in the (safe) rule format

$$\operatorname{AGG}(r) \leftarrow R_1(\underline{\vec{x}_1}, \vec{y}_1) \wedge R_2(\underline{\vec{x}_2}, \vec{y}_2) \wedge \cdots \wedge R_n(\underline{\vec{x}_n}, \vec{y}_n), \quad (2)$$

where r is either a numerical variable or a constant, and AGG is an aggregate operator (e.g., MAX, MIN, SUM, COUNT, AVG); assume  $R_i \neq R_j$  if  $i \neq j$ .

- Given a database instance, let f<sup>+</sup>() and f<sup>-</sup>() be, respectively, the greatest and smallest values of f() across all repairs.
- Aggregate logic L<sub>aggr</sub> [Hella et al., 2001]: FOL + aggregation.
   Question in [Fuxman, 2007] and [Dixit and Kolaitis, 2022]:
   When can f<sup>+</sup>() and f<sup>-</sup>() be expressed in Course?
  - 1.  $f^+()$  and  $f^-()$  are not expressible in  $\mathcal{L}_{aggr}$  if the attack graph of (2) is cyclic (because queries in  $\mathcal{L}_{aggr}$  are Hanf-local).
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Rewriting Example

 $MAX(z) \leftarrow ACTORS(\underline{x}, F, z).$ 

Upper bound rewriting:

 $\mathsf{UB}(\mathtt{MAX}(z)) \leftarrow \mathsf{ACTORS}(\underline{x},\mathsf{F},z)$ 

Lower bound rewriting:

 $\begin{aligned} & \mathsf{POSSIBLE}_{\mathsf{M}}(x) \leftarrow \mathsf{ACTORS}(\underline{x},\mathsf{M},z) \\ & \mathsf{CERTAIN}_{\mathsf{F}}(x,z) \leftarrow \mathsf{ACTORS}(\underline{x},\mathsf{F},z), \neg \mathsf{POSSIBLE}_{\mathsf{M}}(x) \\ & \mathsf{L}(x,\mathtt{MIN}(z)) \leftarrow \mathsf{CERTAIN}_{\mathsf{F}}(\underline{x},z) \\ & \mathsf{LB}(\mathtt{MAX}(z)) \leftarrow \mathsf{L}(x,z) \end{aligned}$ 

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## Counting

Given a Boolean query Q, define the following counting problem:

#### Problem $\problem \problem \p$

Input: A database instance that may violate primary-key constraints.

Question: How many repairs of satisfy Q?

Complexity Classification Task

Input: A self-join-free Boolean conjunctive query Q.

Task: Determine lower and upper complexity bounds on the complexity of #CERTAINTY(q), in terms of common complexity classes like FP and #P.

 Solved in [Maslowski and W., 2013] and generalized to FDs in [Calautti et al., 2022].

Same problem as query answering in block-independent disjoint (BID) probabilistic databases under the restriction that in every block **b**, every tuple has probability <sup>1</sup>/<sub>|b|</sub>.

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## **BID Databases**

Every input to CERTAINTY(Q) is a block-independent disjoint database without probabilities (or with uniform probabilities).

 $^{
m III}$  Inconsistency is not only a burden, but also a chance.  $^1$ 

<u>Name</u> Affiliation P	
$t_1^1$ Fred U. Washington $p_1^1 = 0.3$	
$t_1^2$ U. Wisconsin $p_1^2 = 0.2$	
$t_1^3$ Y! Research $p_1^3 = 0.5$	
$t_2^1$ Sue U. Washington $p_2^1 = 1.0$	
$t_3^1$ John U. Wisconsin $p_3^1 = 0.7$	
$t_3^2$ U. Washington $p_3^2 = 0.3$	
$t_4^1$ Frank Y! Research $p_4^1 = 0.9$	
$t_4^2$ M. Research $p_4^2 = 0.1$	

<sup>&</sup>lt;sup>1</sup>Inspired by [Kern-Isberner and Lukasiewicz, 2017]. The image is from [Dalvi et al., 2009].

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## **Concluding Remarks**

. . .

Consistent Query Answering is an active research area since [Arenas et al., 1999]:

- Database repairing w.r.t. different classes of constraints
- Database repairing and data exchange
- Database repairing and approximations
- Database repairing and preferences
- Database repairing and implementations
- Database repairing and database management systems
- Consistent query answering for queries with negation
- Consistent query answering in description logics
- Consistent query answering over graph databases



## Communications of the ACM, March 2024

(E) Check for

### research

#### Deploying possible world semantics and the challenge of computing the certain answers to queries. BY BENNY KIMELFELD AND PHOKION G. KOLAITIS A Unifying Framework for Incompleteness, Inconsistency, and Uncertainty in Databases

DATABASES ARE OFTEN assumed to have definite content. The reality, though, is the database at hand may be deficient due to missing, invalid, or uncertain information. As a simple illustration, the may invol computat rors in sta models, c that even in full co to queries may depe rectify it: may vary even tho equally leg In the tions, the research ( tal approp deficiency trariness. article is t in which has been proach to ciency in approach. need to r missing records (h since ma and since correct on cific one. I database :



 Possible queries c uncertai viewed a their pos answers true in e
 Queries 1 tractable semantik

# Thanks!

FYI, Brad Pitt celebrated his 60th birthday on December 18, 2023.

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