# Consistent Query Answering with Respect to Primary Keys <br> [Some] Past Research and Future Challenges 

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Complexity of CERTAINTY $(Q)$
$\operatorname{CERTAINTY}(Q)$ in Linear Time (and in FO)

Alternative Semantics

Concluding Remarks

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## Inconsistent Data

## := Perrey Reeves

$\underline{\text { Article Talk } \quad \text { Read Edit View history }}$

From Wikipedia, the free encyclopedia

Perrey Reeves (bor 1970 or 1971 (age 52-53)) ${ }^{11}$ is an American film and televisiolmaetrocs sheris Dest known for her recurring role as Melissa Gold on the television series Entourage from 2004 to 2011 and Marissa Jones in the 2003 comedy Old School.

## Early life [edit]

Reeves was born in New York City and raised in New Hampshire, ${ }^{[2]}$ the daughter of Dr. Alexander Reeves, a


## Inconsistent Databases

> ACTORS $\begin{aligned} & \text { Name Gender Age }\end{aligned}$ Jolie F 48
> $\overline{\bar{P}_{\text {itt }}}{ }^{----\bar{M}}{ }^{---\overline{59}}$
> Pitt _ _ M _ 60

Every actor has, at most, one gender and one age: ACTOR.S PRTMARY KEY (Name)

Data cleaning takes time (and money). Can we already obtain "reliable" information by querving the inconsistent database?

## Inconsistent Databases



Every actor has, at most, one gender and one age: ACTORS PRIMARY KEY (Name).

Data cleaning takes time (and money). Can we already obtain "reliable" information by querying the inconsistent database?

## Querying Inconsistent Databases

For ease of presentation, all queries return a Boolean (true/false).

ACTORS | Name | Gender | Age |  |
| :--- | :--- | :--- | :--- |
|  | $\underline{\text { Jolie }}$ | F | 48 |
| $\overline{\text { Pitt }}$ | - | M | $59^{-}$ |
| Pitt | M | 60 |  |

- Is Pitt's age 60?

$$
\exists y(\text { ACTORS }(\text { Pitt }, y, 60)) \text { is "possibly false". }
$$

- Is Pitt older than Jolie?

$$
\exists y \exists z \exists v \exists w\binom{\text { ACTORS }(\underline{\text { Pitt, }}, y, z) \wedge}{\text { ACTORS }(\underline{\text { Jolie }}, v, w) \wedge z>w} \text { is "certainly true". }
$$

$\square$

## Querying Inconsistent Databases

For ease of presentation, all queries return a Boolean (true/false).

ACTORS | Name | Gender | Age |  |
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- Is Pitt older than Jolie?
$\exists y \exists z \exists v \exists w\binom{$ ACTORS $(\underline{\text { Pitt }, ~} y, z) \wedge}{$ ACTORS $(\underline{\text { Jolie }, ~} v, w) \wedge z>w}$ is "certainly true".
A block is a maximal set of tuples of the same relation that agree on their primary key (blocks are separated by dashed lines). A repair (or possible world) is obtained by picking a single tuple from each block.
With this notion, "certainly true" means "true in every repair". If 2 ages are stored for $n$ actors, there are at least $2^{n}$ repairs.


## Consistent Query Answering for Primary Keys

Given a Boolean query $Q$, define the following decision problem:

```
Problem CERTAINTY(Q)
    Input: A database instance D that may
                violate primary-key constraints.
    Question: Is Q true in every repair of D?
```


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Given a Boolean query $Q$, define the following decision problem:

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## Example

If $Q_{60}=\exists y(\operatorname{ACTORS}($ Pitt, $y, 60))$, then the answer to CERTAINTY $\left(Q_{60}\right)$ is "no" on our example database.

## Remark

We assume that each relation name has a fixed primary key.
Primary-key positions will be underlined. Primary keys can thus be derived from the query.

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## Solving CERTAINTY $(Q)$

Proposition
$\operatorname{CERTAINTY}(Q)$ is in coNP for first-order queries $Q$.
Proof.
A "'no" certificate is a repair that falsifies $Q$.
$\operatorname{CERTAINTY}\left(Q_{60}\right)$ is in FO, as the following are equivalent for every database instance $D$ : $Q$ is true in every repair of $D$; $D$ satisfies $Q_{60} \wedge \neg \exists y \exists z($ ACTORS $($ Pitt, $, y, z) \wedge(z \neq 60))$.

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## The Good, the Bad and the Ugly



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Theorem ([W., 2010])
For $Q_{\text {bad }}=\exists x \exists y(R(\underline{x}, y) \wedge S(\underline{y}, x))$, the decision problem $\operatorname{CERTAINTY}\left(Q_{\text {bad }}\right)$ is in $\mathrm{P} \backslash$ FO (later, it was proven L -complete).

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Theorem ([Chomicki and Marcinkowski, 2005])
For $Q_{\text {ugly }}=\exists x_{1} \exists x_{2} \exists z\left(\operatorname{ACTORS}\left(\underline{x_{1}}, \mathrm{M}, z\right) \wedge \operatorname{ACTORS}\left(\underline{x_{2}}, \mathrm{~F}, z\right)\right)$, the decision problem CERTAINTY $\left(Q_{\text {ugly }}\right)$ is coNP-complete.

## Research Agenda

- We aim to go beyond the task of determining $\operatorname{CERTAINTY}(Q)$ for individual queries $Q$.

A query $Q$ in the class $\mathcal{C}$
The computational complexty of

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## Research Agenda

- We aim to go beyond the task of determining CERTAINTY $(Q)$ for individual queries $Q$.
- For "reasonable" classes $\mathcal{C}$ of queries, write an algorithm for the following problem:

Complexity Classification Task
Input: A query $Q$ in the class $\mathcal{C}$.
Task: The computational complexity of
CERTAINTY $(Q)$, in terms of complexity classes like FO, P, coNP-complete,...

## Which Query Classes Are "Reasonable"?

- The class of (Boolean) conjunctive queries (a.k.a. Select-Project-Join queries):

$$
\begin{equation*}
\exists \vec{u}\left(R_{1}\left(\underline{\vec{x}_{1}}, \overrightarrow{\vec{y}_{1}}\right) \wedge R_{2}\left(\underline{\vec{x}_{2}}, \overrightarrow{y_{2}}\right) \wedge \cdots \wedge R_{n}\left(\underline{\vec{x}_{n}}, \overrightarrow{y_{n}}\right)\right) . \tag{1}
\end{equation*}
$$

where each $Q_{i}$ is of the form (1).

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\end{equation*}
$$

- The class of disjunctions of conjunctive queries (a.k.a. UCQ queries):

$$
Q_{1} \vee Q_{2} \vee \cdots \vee Q_{m}
$$

where each $Q_{i}$ is of the form (1).

## Which Complexity Classes?



## Classifying CERTAINTY $(Q)$ in $\mathrm{P} /$ coNP-complete is Hard

The above conjecture implies
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## Classifying CERTAINTY $(Q)$ in $\mathrm{P} /$ coNP-complete is Hard

Conjecture
If $Q$ is a disjunction of conjunctive queries, then $\operatorname{CERTAINTY}(Q)$ is in P or coNP-complete.

> Theorem ([Fontaine, 2015])
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Journal of Computer and System Sciences 82 (2016) 347-356


Conservative constraint satisfaction re-revisited
Andrei A. Bulatov ${ }^{1}$

## Is it Easier for Conjunctive Queries?

Conjecture
If $Q$ is of the form $\exists \vec{u}\left(R_{1}\left(\vec{x}_{1}, \vec{y}_{1}\right) \wedge \cdots \wedge R_{n}\left(\underline{\vec{x}_{n}}, \overrightarrow{y_{n}}\right)\right)$, then $\operatorname{CERTAINTY}(Q)$ is in P or coNP-complete.

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The above conjecture holds under the assumption that $R_{i} \neq R_{j}$ whenever $i \neq j$.
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The above conjecture holds under the assumption that $n=2$.

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## Is it Easier for Conjunctive Queries?

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If $Q$ is of the form $\exists \vec{u}\left(R_{1}\left(\vec{x}_{1}, \vec{y}_{1}\right) \wedge \cdots \wedge R_{n}\left(\underline{\vec{x}_{n}}, \overrightarrow{y_{n}}\right)\right)$, then CERTAINTY $(Q)$ is in P or coNP-complete.

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The above conjecture holds under the assumption that $R_{i} \neq R_{j}$ whenever $i \neq j$.
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Theorem ([Padmanabha et al., 2023])
The above conjecture holds under the assumption that $n=2$.
Theorem ([Koutris et al., 2021])
The above conjecture holds for queries of the form $\exists x_{1} \cdots \exists x_{n+1}\left(R_{1}\left(\underline{x_{1}}, x_{2}\right) \wedge R_{2}\left(\underline{x_{2}}, x_{3}\right) \wedge \cdots \wedge R_{n}\left(\underline{x_{n}}, x_{n+1}\right)\right)$.

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## The Good Among the Good, the Bad and the Ugly

A directed graph, called attack graph, is defined for every conjunctive query.


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A directed graph, called attack graph, is defined for every conjunctive query.

Theorem ([Koutris and W., 2017])
Let $Q=\exists \vec{u}\left(R_{1}\left(\underline{\vec{x}_{1}}, \overrightarrow{y_{1}}\right) \wedge \cdots \wedge R_{n}\left(\underline{\vec{x}_{n}}, \overrightarrow{y_{n}}\right)\right)$ with $R_{i} \neq R_{j}$ for $i \neq j$. Then,

- if Q's attack graph is acyclic, then CERTAINTY $(Q)$ is in FO;
- if $Q$ 's attack graph is cyclic, then CERTAINTY $(Q)$ is L-hard.

Attack Graph

$$
\begin{aligned}
N^{+} & =\{v\} \\
P^{+} & =\{x\} \\
R_{1}^{+} & =\{y, x, z, r, u\} \\
R_{2}^{+} & =\{y, x, z, r, u\} \\
S^{+} & =\{y, x, u\} \\
U^{+} & =\{y, x, z, r\} \\
T^{+} & =\{x, z, y, u\} \\
W^{+} & =\{u, w\}
\end{aligned}
$$

$S^{+}$, e.g., is the closure of $S^{\prime}$ s key w.r.t. all other FDs.
$S$ can attack with $z \notin S^{+}$.

## Attack Graph and (Consistent) First-Order Rewriting



We construct a first-order formula $\varphi_{N}$ such that for every database:

$$
\varphi_{N} \text { is true in the database } \Longleftrightarrow Q \text { is true in every repair. }
$$

where $\varphi_{P}(v, x)$ is a rewriting of the conjunctive query whose atoms are the atoms of $Q$ except $N(v, x)$, in which variables $v$ and $x$ are free.

## Attack Graph and (Consistent) First-Order Rewriting



We construct a first-order formula $\varphi_{N}$ such that for every database:

## $\varphi_{N}$ is true in the database $\Longleftrightarrow Q$ is true in every repair.

$$
\varphi_{N}:=\exists v\left(\exists x(N(\underline{v}, x)) \wedge \neg \exists x\left(N(\underline{v}, x) \wedge \neg \varphi_{P}(v, x)\right)\right),
$$

where $\varphi_{P}(v, x)$ is a rewriting of the conjunctive query whose atoms are the atoms of $Q$ except $N(\underline{v}, x)$, in which variables $v$ and $x$ are free.
The empty query rewrites to true.

## Attack Graph $\neq$ Join Tree



The subgraph induced by atoms that contain $x$ is not connected.

Attack Graph that Is a Join Tree


Moreover, every internal node $V$ has zero indegree in the attack graph of the subquery rooted at $V(V \in\{P, S, U\})$. Such a join tree is called a Pair-Pruning Join Tree (PPJT) Yannakakis' algorithm extends to the inconsistent setting:

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Theorem ([Fan et al., 2023])
If $Q$ has a PPJT, then CERTAINTY $(Q)$ is in LIN (i.e., problems solvable in linear time).

## Yannakakis+Pruning



$$
\begin{aligned}
T^{\text {join }}(x, z) & \leftarrow T(\underline{x, z}, r) \\
W^{\text {join }}(u) & \leftarrow W(\underline{(u, w})
\end{aligned}
$$

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## Yannakakis+Pruning



$$
\begin{aligned}
S^{\text {join }}(x, y) & \leftarrow S(\underline{y}, x, z) \wedge \neg S^{\text {fadingkey }}(y) \\
S^{\text {fadingkey }}(y) & \leftarrow S(\underline{y}, x, z) \wedge S\left(\underline{y}, x^{\prime}, z\right) \wedge x \neq x^{\prime} \\
S^{\text {fadingkey }}(y) & \leftarrow S(\underline{y}, x, z) \wedge \neg T^{\text {join }}(x, z)
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$$
R_{i}^{\text {join }}(x, y) \leftarrow R_{i}(\underline{y}, x) \wedge \neg R_{i}^{\text {fadingkey }}(y)
$$

$$
R_{i}^{\text {fadingkey }}(y) \leftarrow R_{i}(\underline{y}, x) \wedge R_{i}\left(\underline{y}, x^{\prime}\right) \wedge x \neq x^{\prime}
$$

$$
(1 \leq i \leq 2)
$$

## Yannakakis+Pruning



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U^{\text {join }}(y) & \leftarrow U(\underline{y}, u) \wedge \neg U^{\text {fadingkey }}(y) \\
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$$
R_{i}^{\text {join }}(x, y) \leftarrow R_{i}(\underline{y}, x) \wedge \neg R_{i}^{\text {fadingkey }}(y)
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$$
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## Yannakakis+Pruning



$$
T^{\mathrm{join}}(x, z) \leftarrow T(\underline{x, z}, r)
$$

$$
W^{\text {join }}(u) \leftarrow W(\underline{u, w})
$$

$$
\begin{aligned}
\text { Answer }(y \text { yes }) & \leftarrow N(\underline{v}, x) \wedge \neg N^{\text {fadingkey }}(v) \\
N^{\text {fadingkey }}(v) & \leftarrow N(\underline{v}, x) \wedge \neg P^{\text {join }}(x) \\
P^{\text {join }}(x) & \leftarrow P(\underline{x}, y) \wedge \neg P^{\text {fadingkey }}(x) \\
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(1 \leq i \leq 2) &
\end{aligned}
$$

## Observation Regarding Correctness

$R_{i}$-blocks of size $\geq 2$ can be ignored. For example,

$$
R_{1}\left|\begin{array}{cc}
\underline{y} & x \\
\hline a & c_{1} \\
\underline{a} & c_{2}
\end{array} \quad R_{2}\right| \begin{array}{ll}
\underline{y} & x \\
\hline a & c_{1} \\
\underline{a} & -\underline{c_{2}} \\
\hline- & -
\end{array}
$$

To construct a repair that falsifies the query, pick $R_{1}\left(\underline{a}, c_{i}\right)$ and $R_{2}\left(\underline{a}, c_{j}\right)$ such that $c_{i} \neq c_{j}$.

## LinCQA

- LinCQA is a system that takes as input any query with a PPJT and outputs rewritings in both SQL and non-recursive Datalog with negation.
- https://github.com/xiatingouyang/LinCQA/
- See [Fan et al., 2023] for experiments.



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## Range Consistent Query Answering [Arenas et al., 2001]

For queries returning numbers instead of Booleans.
For ease of presentation, all queries return a single number.

- The lowest answer across all repairs is $\operatorname{MAX}(\{48,52\})=52$;
- the greatest answer across all repairs is $\operatorname{MAX}(\{48,59,53\})=59$;
$\rightarrow$ the interval $[52,59]$ is called the range consistent answer


## Range Consistent Query Answering [Arenas et al., 2001]

For queries returning numbers instead of Booleans.
For ease of presentation, all queries return a single number.

## ACTORS

| Name | Gender | Age |
| :---: | :---: | :---: |
| Jolie | F | 48 |
| $\overline{\text { Pitt }}$ | F | 59 |
| Pitt | M | 60 |
| $\bar{R}$ | F | 52 |
| Reeves | F | 53 |

Get the age of the oldest actress:

$$
\operatorname{MAX}(z) \leftarrow \operatorname{ACTORS}(\underline{x}, F, z)
$$



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For queries returning numbers instead of Booleans.
For ease of presentation, all queries return a single number.

| ACTORS |  |  |
| :---: | :---: | :---: |
| Name | Gender | Age |
| Jolie | F | 48 |
| $\overline{\text { Pitt }}$ | F |  |
| Pitt | M |  |
| $\bar{R}$ | F |  |
| Reeves | F |  |

Get the age of the oldest actress:

$$
\operatorname{MAX}(z) \leftarrow \operatorname{ACTORS}(\underline{x}, F, z)
$$

- The lowest answer across all repairs is $\operatorname{MAX}(\{48,52\})=52$;
- the greatest answer across all repairs is $\operatorname{MAX}(\{48,59,53\})=59$;
- the interval $[52,59]$ is called the range consistent answer.


## Formal Setting

- Numerical terms $f()$ expressible in the (safe) rule format

$$
\begin{equation*}
\operatorname{AGG}(r) \leftarrow R_{1}\left(\underline{\vec{x}_{1}}, \overrightarrow{\hat{y}_{1}}\right) \wedge R_{2}\left(\underline{\vec{x}_{2}}, \overrightarrow{y_{2}}\right) \wedge \cdots \wedge R_{n}\left(\underline{\vec{x}_{n}}, \vec{y}_{n}\right), \tag{2}
\end{equation*}
$$

where $r$ is either a numerical variable or a constant, and AGG is an aggregate operator (e.g., MAX, MIN, SUM, COUNT, AVG); assume $R_{i} \neq R_{j}$ if $i \neq j$.

- Given a database instance, let $f^{+}()$and $f^{-}()$be, respectively, the greatest and smallest values of $f()$ across all repairs.


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- Given a database instance, let $f^{+}()$and $f^{-}()$be, respectively, the greatest and smallest values of $f()$ across all repairs.
- Aggregate logic $\mathcal{L}_{\text {aggr }}$ [Hella et al., 2001]: FOL + aggregation.
- Question in [Fuxman, 2007] and [Dixit and Kolaitis, 2022]:

When can $f^{+}()$and $f^{-}()$be expressed in $\mathcal{L}_{\text {aggr }}$ ?

1. $f^{+}()$and $f^{-}()$are not expressible in $\mathcal{L}_{\text {aggr }}$ if the attack graph of (2) is cyclic (because queries in $\mathcal{L}_{\text {aggr }}$ are Hanf-local).
2. Does the converse hold?

## Rewriting Example

$$
\operatorname{MAX}(z) \leftarrow \operatorname{ACTORS}(\underline{x}, F, z)
$$

- Upper bound rewriting:

$$
\mathrm{UB}(\operatorname{MAX}(z)) \leftarrow \operatorname{ACTORS}(\underline{x}, \mathrm{~F}, z)
$$

- Lower bound rewriting:
$\operatorname{POSSIBLE} \mathrm{M}(x) \leftarrow \operatorname{ACTORS}(\underline{x}, \mathrm{M}, z)$
CERTAIN_F $(x, z) \leftarrow \operatorname{ACTORS}(\underline{x}, \mathrm{~F}, z), \neg$ POSSIBLE $M(x)$
$\mathrm{L}(x, \operatorname{MIN}(z)) \leftarrow$ CERTAIN_F $(\underline{x}, z)$
$\mathrm{LB}(\operatorname{MAX}(z)) \leftarrow \mathrm{L}(x, z)$


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## Concluding Remarks

## Counting

Given a Boolean query $Q$, define the following counting problem:

## Problem $\# C E R T A I N T Y(Q)$

Input: A database instance that may violate primary-key constraints.
Question: How many repairs of satisfy $Q$ ?
$\qquad$

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Given a Boolean query $Q$, define the following counting problem:

## Problem $\sharp$ CERTAINTY $(Q)$

Input: A database instance that may violate primary-key constraints.
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Complexity Classification Task
Input: A self-join-free Boolean conjunctive query $Q$.
Task: Determine lower and upper complexity bounds on the complexity of $\sharp$ CERTAINTY $(q)$, in terms of common complexity classes like FP and $\sharp P$.

## Counting

Given a Boolean query $Q$, define the following counting problem:

## Problem $\sharp C E R T A I N T Y(Q)$ <br> Input: A database instance that may violate primary-key constraints. <br> Question: How many repairs of satisfy $Q$ ?

## Complexity Classification Task

Input: A self-join-free Boolean conjunctive query $Q$.
Task: Determine lower and upper complexity bounds on the complexity of $\sharp$ CERTAINTY $(q)$, in terms of common complexity classes like FP and $\sharp P$.

- Solved in [Maslowski and W., 2013] and generalized to FDs in [Calautti et al., 2022].
- Same problem as query answering in block-independent disjoint (BID) probabilistic databases under the restriction that in every block $\mathbf{b}$, every tuple has probability $\frac{1}{|\mathbf{b}|}$.


## BID Databases

Every input to CERTAINTY $(Q)$ is a block-independent disjoint database without probabilities (or with uniform probabilities).

喚 Inconsistency is not only a burden, but also a chance. ${ }^{1}$

| Researchers: |  |  |  |
| :---: | :---: | :---: | :---: |
| $t_{1}^{1}$$t_{1}^{2}$$t_{1}^{3}$$t_{2}^{1}$$t_{3}^{1}$$t_{3}^{2}$$t_{4}^{1}$$t_{4}^{2}$ | Name | Affiliation | P |
|  | Fred | U. Washington | $p_{1}^{1}=0.3$ |
|  |  | U. Wisconsin | $p_{1}^{2}=0.2$ |
|  |  | Y! Research | $p_{1}^{3}=0.5$ |
|  | Sue | U. Washington | $p_{2}^{1}=1.0$ |
|  | John | U. Wisconsin | $p_{3}^{1}=0.7$ |
|  |  | U. Washington | $p_{3}^{2}=0.3$ |
|  | Frank | Y! Research | $p_{4}^{1}=0.9$ |
|  |  | M. Research | $p_{4}^{2}=0.1$ |

[^0]
## Table of Contents

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## Concluding Remarks

Consistent Query Answering is an active research area since [Arenas et al., 1999]:

- Database repairing w.r.t. different classes of constraints
- Database repairing and data exchange
- Database repairing and approximations
- Database repairing and preferences
- Database repairing and implementations
- Database repairing and database management systems
- Consistent query answering for queries with negation
- Consistent query answering in description logics
- Consistent query answering over graph databases
- ...


## RECORD

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## Database Principles

Foundations of Query Answering on Inconsistent Databases
Jef Wijsen
Available in: PDF
Published in September 2019 (Vol. 48 No.3)

## Communications of the ACM, March 2024

## research

Deploying possible wortd semantics and the
challenge of computing the certain answers to
queries.

## Thanks!

FYI, Brad Pitt celebrated his 60th birthday on December 18, 2023.

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[^0]:    ${ }^{1}$ Inspired by [Kern-Isberner and Lukasiewicz, 2017]. The image is from [Dalvi et al., 2009].

